

**GRID SENSITIVITY CAPABILITY FOR LARGE SCALE STRUCTURES**

**by**

**Gopal K. Nagendra\***

**and**

**David V. Wallerstein\*\***

---

\* Senior Engineer, The MacNeal-Schwendler Corporation, Los Angeles, California.

\*\* Chief, Development, The MacNeal-Schwendler Corporation, Los Angeles, California.

## **ABSTRACT**

This paper presents the considerations and the resultant approach used to implement design sensitivity capability for grids into a large scale, general purpose finite element system (MSC/NASTRAN). The design variables are grid perturbations with a rather general linking capability. Moreover, shape and sizing variables may be linked together. The design is general enough to facilitate geometric modeling techniques for generating design variable linking schemes in an easy and straightforward manner.

Test cases have been run and validated by comparison with the overall finite difference method. The linking of a design sensitivity capability for shape variables in MSC/NASTRAN with an optimizer would give designers a powerful, automated tool to carry out practical optimization design of real life, complicated structures.

## **INTRODUCTION**

This paper presents the considerations and the resultant approach used to implement design sensitivity capability for grids into MSC/NASTRAN. MSC/NASTRAN is a large-scale, general purpose computer program which solves a wide variety of engineering problems by the finite element method. In 1983, the design sensitivity analysis (DSA) capability was installed in MSC/NASTRAN. This capability has recently been enhanced to include a fully integrated optimization capability for sizing variables. With the addition of shape sensitivity capability and with the increasing interest in aerospace and automotive industries, this general capability can be used in its own right for carrying out sensitivity analysis of complicated real life structures.

Shape optimization is still in a state where fundamental research is needed (Reference 1). The integration of shape optimization concepts within Finite Element Methods (FEM) and Computer Aided Designs (CAD) should help to bridge the gap between these two technologies. To be successful, the proposed integration of software should lead to a system easy to use. From a practical point of view, the computational tool should indeed be employed by design engineers with only a superficial knowledge of the theoretical basis of each technique.

The following sections will be devoted to the basic procedure for an efficient design variable linking scheme using reduced basis concepts. These are necessary to avoid generating unrealistic designs due to independent node movements.

The next section provides an overview of the design sensitivity capability in MSC/NASTRAN. The design constraints can be weight, volume, frequency, buckling loads, displacements, stresses, strains or forces. The design variables can be shape or sizing variables. The method chosen is a semianalytical approach based on a first variation (finite difference scheme) of the system's equilibrium equations with respect to the design variables.

A brief description follows of the program architecture and considerations that go into implementing such a capability into a large-scale, general-purpose computer program. The important considerations are ease of use, generality, compatibility with the existing architecture, data organization and management and, finally, a good restart capability for an active man/machine interaction.

Errors associated with shape sensitivity analysis using the semianalytical approach as shown in Reference 2 are examined. An iterative scheme and error index are employed to minimize the errors in the sensitivity calculations.

Two example problems were chosen to validate the capability and to highlight some of the salient features. The first example problem is a beam modeled by solid elements, with x-section as design variable. The second example problem is a cantilever beam with length as design variable.

## BASIC PROCEDURE

When dealing with shape optimization problems, the design variables must be selected very carefully. The coordinates of the boundary nodes of the finite element model is a straightforward choice. This choice, however, exhibits many severe drawbacks. The set of design variables is very large and the cost and difficulty of the minimization process increase. It has a tendency to generate unrealistic designs due to the independent node movement and additional constraints avoiding such designs are difficult to cope with. Moreover, an automatic mesh generator is necessary to maintain the mesh integrity throughout the optimization process. One obvious remedy is to avoid a one-to-one correspondence between the finite element model and the design variables.

One way to achieve this goal is to use the concept of "design model" utilizing "reduced basis vectors". The general form of this relationship is (Reference 3)

$$\{\Delta g\} = [T] \{x\} \quad (1)$$

$\begin{matrix} nx1 & nxm & mx1 \end{matrix}$

The given design  $\Delta g$  define a system of  $n$  variables. We refer to columns of  $T$  as basis vectors. Clearly the method is most useful if  $m \ll n$  and the method will produce a true optimum only if some combination of basis vectors can define that optimum.

From Equation 1, it is obvious that the reduced basis approach is simply a design variable linking scheme where the basis vectors are columns of the coefficient matrix T. Ideally, the basis vectors could most conveniently be generated using a graphics preprocessor.

To clarify ideas, consider the coordinate update equation given by

$$\begin{Bmatrix} x_{new} \\ y_{new} \\ z_{new} \end{Bmatrix} = \begin{Bmatrix} x_{old} \\ y_{old} \\ z_{old} \end{Bmatrix} + \sum x \cdot \begin{Bmatrix} D\text{-cosine-x} \\ D\text{-cosine-y} \\ D\text{-cosine-z} \end{Bmatrix} + \begin{Bmatrix} C_x \\ C_y \\ C_z \end{Bmatrix} \quad (2)$$

The constant term vector  $C_x, C_y, C_z$  are computed to make necessary adjustments so that  $x_{new}, y_{new}, z_{new}$  are equal to  $x_{old}, y_{old}, z_{old}$  for the initial values of the design variables  $x$  supplied by the user. Notice that in Equation 2, for the initial configuration, the values of the initial design variables  $x$  supplied by the user need not be zero. Equation 2 may be rewritten as

$$\begin{Bmatrix} x_{new} \\ y_{new} \\ z_{new} \end{Bmatrix} - \begin{Bmatrix} x_{old} \\ y_{old} \\ z_{old} \end{Bmatrix} - \begin{Bmatrix} C_x \\ C_y \\ C_z \end{Bmatrix} = \sum x \cdot \begin{Bmatrix} D\text{-cosine-x} \\ D\text{-cosine-y} \\ D\text{-cosine-z} \end{Bmatrix}$$

or

$$\{\Delta g\}_i = [T]_{ij} \{x\}_j$$

this is the same form as shown in Equation 1. The columns of the T matrix are then the basis vectors. The capability exists for the user to input the elements of the T matrix directly or in the future he may generate elements of the T matrix using a preprocessor. It should be clear from the above formulation that the design variables are the grid perturbations and not the grid coordinates.

## DESIGN SENSITIVITY CAPABILITY IN MSC/NASTRAN

Design sensitivity analysis estimates the effects of interrelated design variables such as element properties and materials on the structural response quantities such as displacement, stress, natural frequency, buckling loads - and for composites lamina stresses and failure indices. Design sensitivity coefficients are defined as the gradients of the design constraints with respect to the design variables at the current design point. The method chosen for incorporation into MSC/NASTRAN is a semianalytical approach, based on a first variation (finite difference scheme) of the systems equilibrium equations with respect to the design variables.

Let  $\psi_i(b_j, u)$  be a set of design constraints which are functions of  $b_j$  design variables and displacements  $u$ . The design constraints are expressed as

$$\psi_i(b_j, u) \leq 0$$

The first variation in  $\psi_i$  is given as

$$\delta\psi_i = \underbrace{\left[ \frac{\partial\psi_i}{\partial b_j} \right]}_{ixj} \underbrace{\{\delta b_j\}}_{jx1, \text{ u-fixed}} + \underbrace{\left[ \frac{\partial\psi_i}{\partial u} \right]}_{ixn} \underbrace{\{\delta u\}}_{nx1, \text{ b-fixed}}$$

consider  $u$  as a function of  $b_j$ , then

$$\underbrace{\{\delta u\}}_{nxj} = \underbrace{\left[ \frac{du}{db_j} \right]}_{jx1} \underbrace{\{\delta b_j\}}_{jx1}$$

and, therefore,

$$\{\delta\psi_i\} = \left( \left[ \frac{\partial\psi_i}{\partial b_j} \right] + \left[ \frac{\partial\psi_i}{\partial u} \right] \left[ \frac{du}{db_j} \right] \right) \{\delta b_j\}$$

or

$$\frac{\delta\psi_i}{\delta b_j} \equiv \frac{\Delta\psi_i}{\Delta b_j} = \left[ \frac{\partial\psi_i}{\partial b_j} \right] + \left[ \frac{\partial\psi_i}{\partial u} \right] \left[ \frac{du}{db_j} \right]$$

The matrix  $\frac{du}{db_j}$  can be evaluated by taking the first variation of the systems equilibrium equation,

$$[K] \{u\} = \{P\}$$

which gives

$$[K] \{\Delta u\} + [\Delta K] \{u\} = \{\Delta P\}$$

solving for  $\{\Delta u\}$

$$\{\Delta u\} = [K]^{-1} (\{\Delta P\} - [\Delta K] \{u\})$$

or

$$[\Delta u] = [K]^{-1} (\{ \Delta P(\Delta b_1) \}, \{ \Delta P(\Delta b_2) \}, \dots, \{ \Delta P(\Delta b_j) \}) \\ - [K]^{-1} (\{ \Delta K(\Delta b_1) \} \{u\}, \{ \Delta K(\Delta b_2) \} \{u\}, \dots, \{ \Delta K(\Delta b_j) \} \{u\})$$

The elements of  $\left[ \frac{d\psi_i}{du} \right]$  matrix for an element constraint such as stress, force, or failure index can be expressed by the relationship

$$\{\psi_i\} = [S]_e \{u\}$$

or

$$\left[ \frac{d\psi_i}{du} \right] = [S]_i$$

The design sensitivity coefficient matrices may thus be expressed as

$$[\Lambda_{ij}] \equiv \left( \left\{ \frac{\Delta\psi_i}{\Delta b_1} \right\}, \left\{ \frac{\Delta\psi_i}{\Delta b_2} \right\}, \dots, \left\{ \frac{\Delta\psi_i}{\Delta b_j} \right\} \right) \Big|_{u_{fixed}} \\ + [S]_i \left( \left\{ \frac{\Delta U_1}{\Delta b_1} \right\}, \left\{ \frac{\Delta U_2}{\Delta b_2} \right\}, \dots, \left\{ \frac{\Delta U_j}{\Delta b_j} \right\} \right) \Big|_{b_{fixed}}$$

From this equation it is easy to see that the number of additional case control records (additional loading cases) required for design sensitivity analysis is equal to the number of design variables for each subcase (Design Space Technique).

A typical term of the coefficient matrix may be written as

$$\Lambda_{ij} = \left( \frac{S^{B+\Delta B} u^B}{\Delta B} - \frac{S^B u^B}{\Delta B} \right) + \left( \frac{S^B u^{B+\Delta B}}{\Delta B} - \frac{S^B u^B}{\Delta B} \right)$$

where B represents the base line or original state and B + ΔB represents the perturbed state. The first expression in parentheses on the right-hand side is thus the change in response quantity due to a change in design variable for the original solution vector. The second term represents the change in response quantity due to a change in displacement for the unperturbed design variable. For displacement constraints, the first term in parentheses on the right-hand side is identically zero.

The design constraints can be weight, volume, frequency, buckling loads, displacements, stresses, strains, forces, lamina stresses, lamina strains, failure indices or user supplied synthetic equations. The design variables can be grid movements or properties. The shape and sizing variables may be linked together.

## PROGRAM ARCHITECTURE

In order to understand the reasons behind how a development is introduced into a large finite element program, a knowledge of the program architecture and technical purpose is necessary. A brief description of MSC/NASTRAN is presented as background (Reference 4)

The cornerstone of MSC/NASTRAN's architecture is its Executive System, whose essential functions are to establish and control the sequence of calculations, to allocate files, and to maintain a restart capability. Engineering calculations are performed by approximately 200 Functional Modules which communicate only with the Executive System and not with each other. Flexibility is maintained by a macro-instruction language called DMAP, which is under user control, but which also serves to establish preformatted calculation sequences for the major types of analysis, including linear analysis, buckling, vibration mode analysis, and design sensitivity.

The calculation of finite element data is concentrated exclusively in a few modules. The element matrices for stiffness, structural damping, and differential stiffness for elements of the structural model are generated in the Element Matrix Generator (EMG) module. These element matrices are subsequently assembled to form the elastic stiffness matrix, the structural damping matrix, the mass matrix, or the differential stiffness matrix.

The element contribution to the load vector is generated in the load generator module and the element stress and force are generated in the recovery module. In all these modules, the finite element descriptions are defined in the Element Summary Table (EST). The EST contains the element connection, material property and sectional property information.

Taking advantage of the table driven concept used by the element modules, much of the element dependent development can be avoided in implementing design sensitivity. The reason is that a procedure could be developed which only involves building EST tables that would cause existing modules to form the necessary element data.

How a given capability is introduced into a commercial general purpose finite element program is as important an issue to the user as its theoretical sophistication. If the user views a capability as hard to use, as having an insufficient capacity to solve his problem, or taking an inordinate time to comprehend its output, the product is of little practical use. In addition, the program developer, while heeding the user's needs, has to keep sight of the program as a whole when adding new capabilities. This involves interfacing well with existing capabilities, maintaining program reliability and generality, and producing software that makes effective use of computer resources.

The user interface is a major consideration in the design of a new capability. The following issues were considered when building up the design sensitivity analysis (DSA) capability.

1. DSA input should be straightforward, but allow flexibility to model complex structural design concepts.
2. DSA output should be concise and easily understood.
3. Avoid arbitrary program limits which restrict the allowable element types, constraint quantities, and problem size.
4. Provide an interface for external optimization postprocessors.

A brief discussion of the processes involved in a typical DSA STATIC analysis in MSC/NASTRAN will help bring into perspective the work involved in the various parts of the DSA solution.

DSA in a STATICS analysis is based on solving for  $\{\Delta u\}$  the first order variation of the nodal equilibrium equation:

$$[K^0] \{\Delta u\} = \{\Delta P\} - [\Delta K] \{u^0\}$$

The DSA problem in this paper is considered to be the additional task required after the solution of primary analysis. By restarting from the primary STATIC analysis, the solution of the DSA system equation only involves the calculation of the right-hand side and the backward pass operation in the solution of  $\Delta u$ .

The work involved in solving the system equations (backward pass operation) is a function of the product of the number of design variables and loading conditions. The following DSA tasks are required in addition to solving the system equations:

1. DSA Data Organization
2. DSA Data Assembly
3. DSA Data Recovery

These tasks are functions of the triple product of the number of design variables, design constraints and loading conditions. For large DSA problems, the data organization, assembly and recovery tasks are the dominant users of computer resources.

Another major consideration was to support all structural finite element types in MSC/NASTRAN. Since a large number of the elements developed are semiempirical, the determination of consistent element derivative formulations cannot be practically accomplished. Therefore, a method was developed to calculate element derivatives by a differencing scheme about the current design point. This method involved the calculation of the element matrix at the design point plus or minus the user specified design



variable increment. This element data is differenced with the data at the design point to determine the corresponding element derivatives. For example, the following shows the change in element stiffness due to a change in the design variable.

$$[\Delta K] = [K_{B+\Delta B}] - [K^0]$$

An initial analysis is carried out to identify critical constraints and a data base is created. In the succeeding run, information about constraints, design variables, maximum and minimum side constraints is supplied. A special DMAP package was created which exploits the data base technology.

### ERRORS ASSOCIATED WITH SEMI-ANALYTICAL APPROACH IN SHAPE SENSITIVITY

In Reference 2 it was shown that the semi-analytical approach can have serious accuracy problems for shape design variables in structures modeled by beam, plate, truss, frame and solid elements. An error index was developed to test the accuracy of the semi-analytical approach and some methods were proposed for improving the accuracy of the semi-analytical method. In the following section, the interactive scheme proposed in Reference 2 is examined in greater detail.

Consider the systems equilibrium equation

$$[k] \{u\} = \{P\}$$

Taking the first variation of the systems equilibrium equation gives

$$[k + \Delta k] \{u + \Delta u\} = \{P + \Delta P\}$$

Expanding the above equation and retaining the second order terms gives

$$[k] \{\Delta u\} = \{\Delta P\} - [\Delta k] \{u\} - [\Delta k] \{\Delta u\}$$

which can be cast as an iterative scheme

$$[k] \{\Delta u\}_i = \{\Delta P\} - [\Delta k] \{u\}_0 - [\Delta k] \{\Delta u\}_{i-1} \quad (3)$$

This iterative scheme can be used quite effectively with the error index as shown in Reference 3, i.e.,

$$E^{lmn} = \frac{[\Lambda]^l - [\Lambda]^m}{[\Lambda]^n} \quad (4)$$

An error index of the type described above is almost a must for a large scale system in the optimization context. If the sensitivity derivatives are significantly in error, the program needs to detect it and stop the execution to save time and resources of the user.

## NUMERICAL EXAMPLES

Two example problems were chosen to validate the capability and to highlight some of the salient features.

### Example 1 - Beam using solid elements

A cantilever beam is subjected to a tip loading. The model of the beam is shown in Figure 1. The beam is modeled using solid hexahedron elements. The analysis model consists of five HEXA elements and 24 grids.

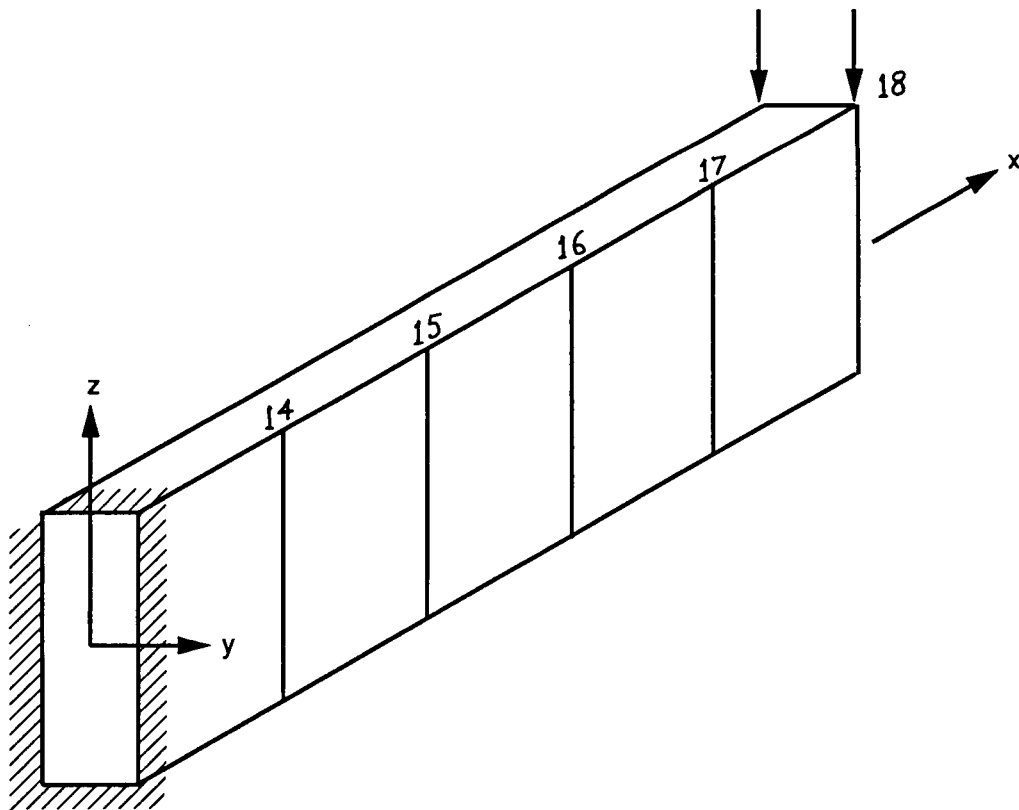
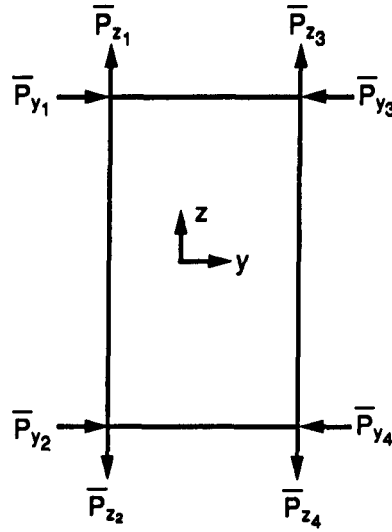


Figure 1. Beam using solid elements.

The design model consists of one design variable to perturb beam cross section to maximize the bending inertia about the y-axis. The cross-section is perturbed as shown in Figure 2 below.



**Figure 2. Design Variable Linking Scheme.**

$P_{y1}$  and  $P_{z1}$  are perturbation vectors in the y and z direction respectively.

The perturbation vectors are given by

$$P_{y1} = P_{y2} = 0.01(0\hat{i} + \hat{j} + 0\hat{k})$$

$$P_{y3} = P_{y4} = 0.01(0\hat{i} - \hat{j} + 0\hat{k})$$

$$P_{z1} = P_{z3} = 0.01(0\hat{i} + 0\hat{j} + \hat{k})$$

$$P_{z2} = P_{z4} = 0.01(0\hat{i} + 0\hat{j} - \hat{k})$$

The sensitivity coefficient results calculated using the semi-analytical approach are compared to the Overall Finite Difference (OFD) approach, wherein the entire problem is solved again for the perturbed configuration. The results are shown in Table 1. As can be seen, the correlation between the semi-analytical approach and the OFD is excellent for this particular example problem.

Grid-ID	Sensitivity OFD	Sensitivity SA	Error
14	-.0313	-.0307	1.92
15	-.0861	-.0845	1.86
16	-.0166	-.0163	1.81
17	-.0235	-.0231	1.7
18	-.0397	-.0389	2.0

Table 1. Vertical displacements along beam length..

### Example 2 - Cantilever beam subjected to end moment

The same cantilever beam that was considered in Reference 2 is taken as an example here. The cantilever beam has uniform rigidity  $EI$  and length  $L$  under a tip moment  $M$  as shown in Figure 3.

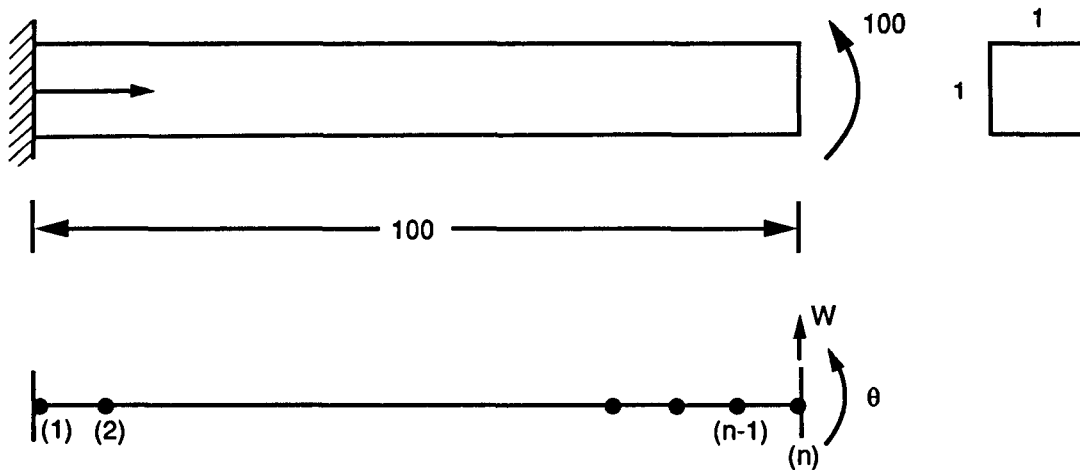


Figure 3. Cantilever beam subjected to end moment.

The modulus of elasticity is taken equal to  $E = 10^7$ . The theoretical tip deflection for the above configuration is

$$\delta = \frac{M L^2}{2EI} \quad (5)$$

The structure has been idealized into 20 beam elements. Sensitivity coefficients have been calculated for the tip displacement with respect to the length of the beam. The design model contains a single design variable, i.e., the length of the beam. The grid perturbations of all the grids have been linked together so that the perturbations vary linearly from root to tip of the beam.

Parametric studies have been carried out to determine the effect of the step size on the error. From Equation 5, the exact results are available.

First the sensitivity analysis is carried out using the Overall Finite Difference (OFD) and the semi-analytical (SA) method for step sizes of 1%, 0.5%, 0.1 % and 0.01%. The results, without using any iterative schemes are shown in Table 2. As can be seen, the results are quite accurate for a step size of 0.01%. Whereas they progressively degrade for increasing values of the step size and become quite unacceptable for a step size of 1%.

In Reference 2, the errors for the beam-type structure are associated with an incompatibility of the sensitivity field with the structural model. The error in the finite difference approximation consists of a truncation error because of neglecting higher order terms in the Taylor series expansion and a condition error because of the limited precision available for the computer. Thus, an optimum value of step-size would minimize the truncation error without the condition error becoming significant. As suggested in Reference 3, central difference scheme is an alternative to iterations.

Step-size (%)	Sensitivity Coefficient $\Lambda$
1.0	-10.459
0.5	-4.716
0.1	0.010
0.01	1.08
0.001	1.1885
0.0001	1.1874
0.00001	0.914*

\* Degrades. (Theoretical value = 1.2)

**Table 2. Variation of sensitivity coefficients with respect to step-size.**

Next, we use the iterative scheme of Equation 3 to converge to the correct solution. As can be seen from Figure 4, the smaller the value of the step size, the lesser the number of iterations required to converge to the correct solution.

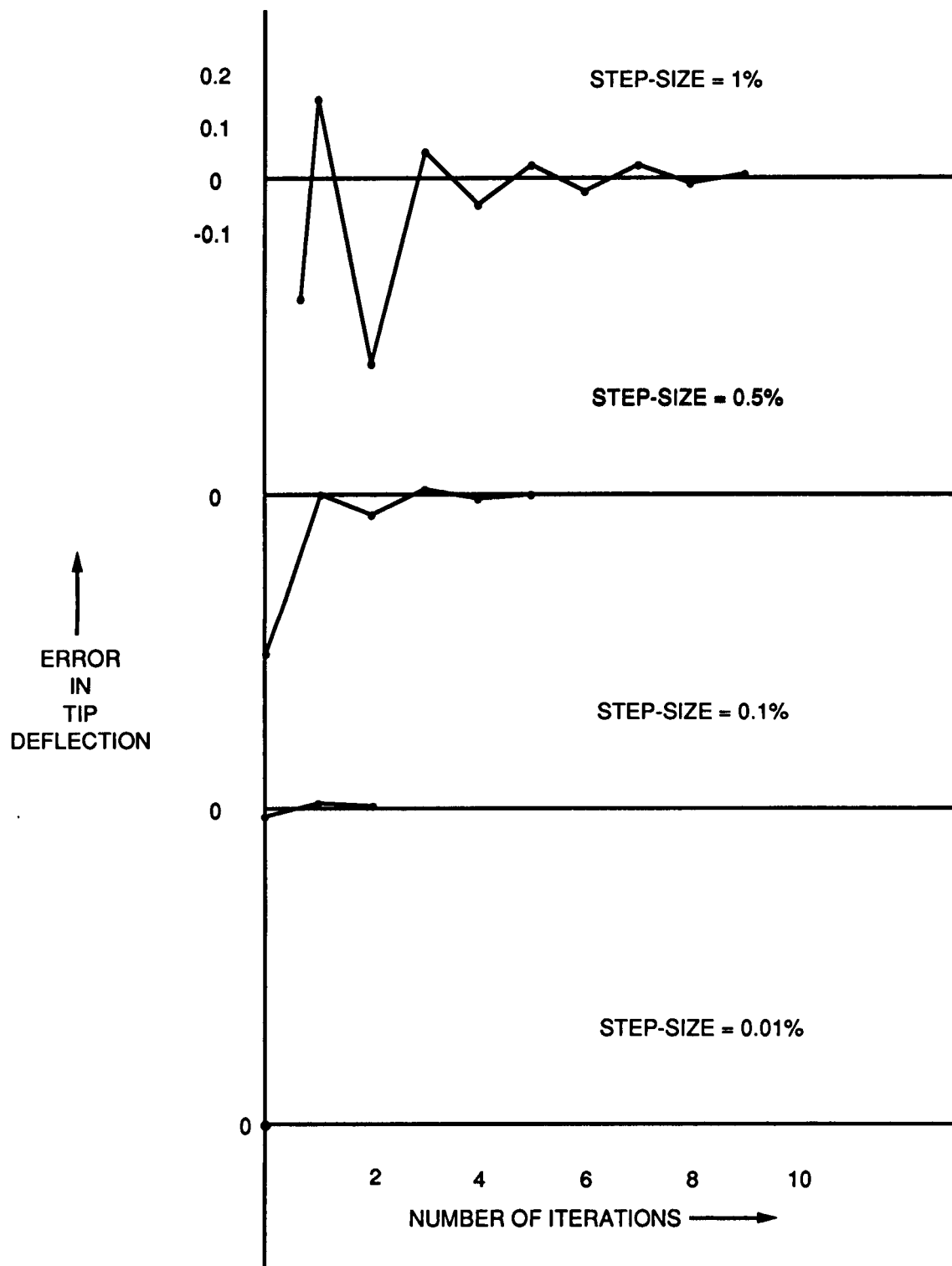


Figure 4. Plot of number of iterations versus error in tip-displacement using SA method.

## CONCLUSIONS

This paper presented the considerations and the resultant approach used to implement design sensitivity capability for grids into a large scale, general purpose finite element system (MSC/NASTRAN). The design variables are grid perturbations with a rather general linking capability. Moreover shape and sizing variables may be linked together. The design is general enough to facilitate geometric modeling techniques for generating design variable linking schemes in an easy and straightforward manner.

The errors shown to be associated with the semianalytic method for shape variables for beam type structures can be mitigated by resorting to an iterative scheme. Examples have been presented highlighting the salient features of the approach.

## REFERENCES

1. Fleury, C. and Braibant, V., *Toward a Reliable C.A.D. Tool*, computer aided optimal design, Troia, Portugal, July 1986
2. Barthelemy, B. and Haftka, R. T., *Accuracy Analysis of the Semi-Analytical Method for Shape Sensitivity Calculation*, 29th SDM Conference, April 18-20/Williamsburg, VA 572-581
3. Vanderplaats, G. N. *Numerical Optimization Techniques for Engineering Design: With Applications*, McGraw-Hill, 1984
4. Nagendra, G-K. and Fleury, C., *Sensitivity and Optimization of Composite Structures using MSC/NASTRAN*, NASA CP-2457, NASA Langley Research Centre, Hampton, Virginia, September 25-26, 1986